# On generalized spaces of persistence diagrams

**Aleksandr Savchenko**
(Kherson State Agrarian University, Stretenska st., 23, Kherson, 73006, Ukraine)
*E-mail:* `savchenko.o.g@ukr.net`

Topological Data Analysis (TDA) is a relative new branch of applied mathematics. It provides some metric and topological structures for analyzing big data represented as point set in the euclidean spaces or more general metric (topological) spaces.

The persistent homology is one of the most important topological tools in TDA (see [1]). In order to visualize the persistent homology one uses the so-called persistence diagrams (see, e.g., [2] and the references therein). The sets of persistence diagrams bear natural metrics or topology and the studying of the metric and topological properties of the obtained spaces is important for understanding properties of the data sets.

Following [3] we consider the generalized spaces of persistence diagrams. Let $X$ be a set. A diagram on $X$ is a function $D\colon X^2 \to \mathbb{Z} = 0$ such that $D(p) = 0$ for all but finitely many $p \in X^2$, and $D(p) = 0$ for all $p \in \Delta_X = \{(x,x) \mid x \in X\} \subset X^2$. In [4] it is remarked that the set $\mathcal{D} = \mathcal{D}(X)$ of all persistence diagrams can be naturally identified with the infinite symmetric product $SP^\infty(X^2/\Delta_X)$ (with the base point $* = \Delta_X$).

If $X$ is a topological space and $X = \varinjlim X_n$, where $X_1 \subset X_2 \subset X_3 \subset \dots$, then one can topologize $\mathcal{D}$ as $\varinjlim SP^n(X_n^2/(\Delta_X \cap X_n^2)$ having in mind a natural identification of $[x_1, \dots, x_n] \in SP^n(X_n^2/(\Delta_X \cap X_n^2)$ with $[x_1, \dots, x_n, *] \in SP^{n+1}(X_{n+1}^2/(\Delta_X \cap X_{n+1}^2)$.

Our result is a generalization of one of the main results of [4]. Recall that $\mathbb{R}^\infty$ is the direct limit of the sequence $\mathbb{R} \subset \mathbb{R}^2 \subset \mathbb{R}^3 \subset \dots$. An $\mathbb{R}^\infty$-manifold is a $k_\omega$-space, which is locally homeomorphic to $\mathbb{R}^\infty$.

An ANR-space is an absolute neighborhood retract in the class of metrizable spaces.

**Теорема 1.** *Let $X = \varinjlim X_n$, where $(X_n)$ is a sequence of finite-dimensional compact metrizable ANR-spaces. If $\dim X > 0$, then the space $\mathcal{D}(X)$ is an $\mathbb{R}^\infty$-manifold.*

One can also find some sufficient conditions on $X$ such that the space $\mathcal{D}(X)$ is a $Q^\infty$-manifold, where $Q$ is the Hilbert cube and $Q^\infty$ is the direct limit of the sequence

$$Q = Q \times \{*\} \hookrightarrow Q \times Q = Q \times Q \times \{*\} \hookrightarrow Q \times Q \times Q = \dots,$$

for an arbitrary $* \in Q$.

The proofs are based on Sakai's Characterization Theorem for $\mathbb{R}^\infty$ and $Q^\infty$ (see [5]).

## Література

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